1 Trees, Graphs, and Traversals, Oh My!
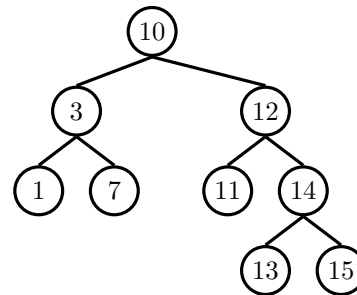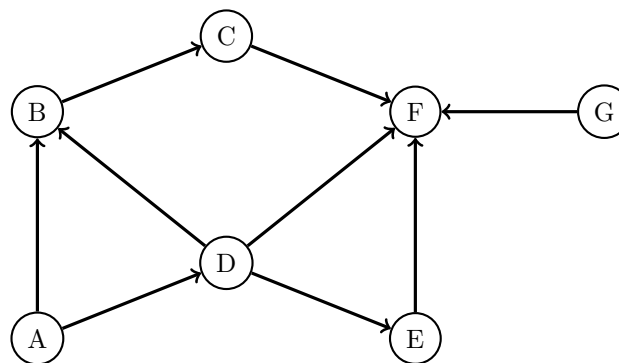
(a) Write the following traversals of the BST below.

Pre-order:

In-order:

Post-order:

Level-order (BFS):

(b) Write the graph below as an adjacency matrix, then as an adjacency list. What would be different if the graph were undirected instead?

(c) Write the order in which (1) DFS pre-order, (2) DFS post-order, and (3) BFS would visit nodes in the same directed graph above, starting from vertex A. Break ties alphabetically.

Pre-order:

Post-order:

BFS:

2   Absolutely Valuable Heaps

(a) Assume that we have a binary min-heap (smallest value on top) data structure called `MinHeap` that has properly implemented the `insert` and `removeMin` methods. Draw the heap and its corresponding array representation after each of the operations below:

```
MinHeap<Character> h = new MinHeap<>();
h.insert('f');
h.insert('h');
h.insert('d');
h.insert('b');
h.insert('c');
h.removeMin();
h.removeMin();
```

(b) Your friendly TA Allen challenges you to create an integer max-heap without writing a whole new data structure. Can you use your min-heap to mimic the behavior of a max-heap? Specifically, we want to be able to get the largest item in the heap in constant time, and add things to the heap in $\Theta(\log n)$ time, as a normal max heap should.

Hint: You should treat the `MinHeap` as a black box and think about how you should modify the arguments/return values of the heap functions.

3   Trinary Search Tree

We'd like a data structure that acts like a BST (Binary Search Tree) in terms of operation runtimes but allows duplicate values. Therefore, we decide to create a new data structure called a TST (Trinary Search Tree), which can have up to three children, which we'll refer to as `left`, `middle`, and `right`. In this setup, we have the following invariants, which are very similar to the BST invariants:

1. Each node in a TST is a root of a smaller TST

2. Every node to the `left` of a root has a value "lesser than" that of the root

3. Every node to the `right` of a root has a value "greater than" that of the root

4. **Every node to the `middle` of a root has a value equal to that of the root**

Below is an example TST to help with visualization.
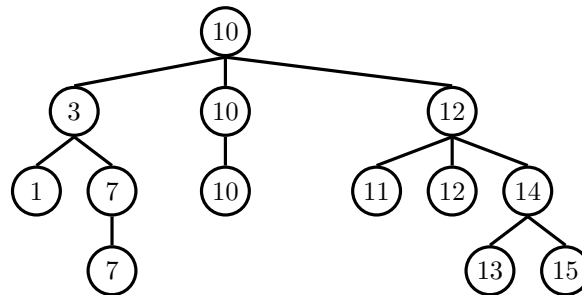
Describe an algorithm that will print the elements in a TST in **descending** order. *(Hint: recall that an in-order traversal for a BST gives elements in increasing order.)*